A DC Algorithm for Solving Multiobjective Stochatic Problem via Exponential Utility Functions

Ramzi Kasri and Fatima Bellahcene

LAROMAD, Mouloud Mammeri University, Faculty of Sciences, BP 17 RP, 15000 Tizi-Ouzou, Algeria.
ramzi.kasri@ummto.dz, bellahcene.fat@gmail.com

Abstract. In this paper we suggest an algorithm for solving a multiobjective stochastic linear programming problem with normal multivariate distributions. The problem is first transformed into a deterministic multiobjective problem introducing the expected value criterion and an utility function. The obtained problem is reduced to a monobjective quadratic problem using a weighting method. This last problem is solved by DC algorithm.

Keywords: Multiobjective programming · Stochastic programming · DCA · DC programming · Utility function · Expected value criterion.

1 Introduction

Multiobjective stochastic linear programming (MOSLP) is a tool for modeling many concrete real-life problems because it is not obvious to have the complete data about problems parameters. Such a class of problems includes investment and energy resources planning [10, 21], manufacturing systems in production planning [7, 8], mineral blending [13], water use planning [2, 5] and multi-product batch plant design [24]. So, to deal with this type of problems it is required to introduce a randomness framework.

In order to obtain the solutions for these multiobjective stochastic problems, it is necessary to combine techniques used in stochastic programming and multiobjective programming. From this, two approaches are considered, both of them involve a double transformation. The difference between the two approaches is the order in which the transformations are carried out. Ben Abdelaziz qualified as *multiobjective approach* the perspective which transform first, the stochastic multiobjective problem into its equivalent multiobjective deterministic problem, and *stochastic approach* the techniques that transform in first the stochastic multiobjective problem into a monobjective stochastic problem [4].

Several interactive methods for solving (MOSLP) problems have been developed. We can mention the Probabilistic Trade-off Development Method or PROTRADE by Goicoechea et al. (1976) [11], The Strange method proposed by

Teghem et al. (1986) [22] and the interactive method with recourse which uses a two stage mathematical programming model by Klein et al. (1990) [12].

In this paper, we propose another approach which is a combination between the *multiobjective approach* and a nonconvex technique (Difference of Convex functions), to solve the multiobjective stochastic linear problem with normal multivariate distributions. The DC programming and DC Algorithm have been introduced by Pham Dinh Tao in 1985 and developed by Le Thi Hoai An and Pham Dinh Tao since 1994 [14–17]. This method has proved its efficiency in a large number of noncovex problems [18–20].

The paper is structured as follows: In section 2, the problem formulation is given. Section 3, shows how to reformulate the problem by introducing utility functions and applying the weighting method. Section 4 presents a review of DC programming and DCA. Section 5 illustrates the application of DC programming and DCA for the resulting quadratic problem. Our, experimental results are presented in the last section.

2 Problem statement

Let us consider the multiobjective stochastic linear programming problem formulated as follows:

$$\begin{array}{c} \min\ (\tilde{c}_1 x, \tilde{c}_2 x, ..., \tilde{c}_q x), \\ s.t.\ \ x \in S, \end{array} \tag{1}$$

where $x = (x_1, x_2, ..., x_n)$ denotes the n-dimensional vector of decision variables. The feasible set S is a subset of n-dimensional real vector space $\mathbb{R}^n$ characterized by a set of constraint inequalities of the form $Ax \leq b$; where A is an $m \times n$ coefficient matrix and b an m-dimensional column vector. We assume that S is nonempty and compact in $\mathbb{R}^n$. Each vector $\tilde{c}_k$ follows a normal distribution with mean $\overline{c}_k$ and covariance matrix V_k. Therefore, every objective $\tilde{c}_k x$ follows a normal distribution with mean $\mu_k = \overline{c}_k x$ and variance $\sigma_k^2 = x^t V_k x$.

In the following section, we will be mainly interested in the main way to transform problem (1) into an equivalent multiobjective deterministic problem which in turn will be reformulated as a DC programming problem.

3 Transformations and Reformulation

First, we will take into consideration the notion of risk. Assuming that decision makers' preferences can be represented by utility functions, under plausible assumptions about decision makers's risk attitudes, problem (1) is interpreted as:

$$\begin{array}{c} \min\limits_{x}(E[U(\tilde{c}_1 x)], E[U(\tilde{c}_2 x)], ..., E[U(\tilde{c}_q x)]), \\ s.t.\ \ x \in S. \end{array} \tag{2}$$

The utility function U is generally assumed to be continuous and convex. In this paper, we consider an exponential utility function of the form $U(r) = 1 - e^{-ar}$, where r is the value of the objective and a the coefficient of incurred risk (a

large corresponds to a conservative attitude). Our choice is motivated by the fact that exponential utility functions will lead to an equivalent quadratic problem which encouraged us to design a DC method to solve it simply and accurately. Therefore, if $r \sim N(\mu, \sigma^2)$, we have:

$$E(U(r)) = \int_{-\infty}^{+\infty} (1 - e^{-ar}) \frac{e^{-(r-\mu)^2/2\sigma^2}}{\sqrt{2\pi}} \frac{dr}{\sigma} = 1 - e^{\frac{\sigma^2 a^2}{2} - \mu a}.$$

Minimizing $E(U(r))$ means maximizing $\frac{\sigma^2 a^2}{2} - \mu a$ or minimizing $\mu - \frac{\sigma^2 a}{2}$.

Our aim is to search for efficient solutions of the multiobjective deterministic problem (2) according to the following definition:

Definition 1. *[3] A feasible solution x^* to problem (1) is an efficient solution if there is not another feasible x such that $E[U(\tilde{c}_k x))] \geq E[U(\tilde{c}_k x^*)]$ with at least one strict inequality. The resulting criterion vector $E[U(\tilde{c}_k x^*)]$ is said to be non-dominated.*

Applying the widely used method for finding efficient solutions in multiobjective programming problems, namely the weighting sum method [3, 6], we assign to each objective function in(2) a non-negative weight w_k and aggregate the objectives functions in order to obtain a single function. Thus, problem (2) is reduced to:

$$\begin{aligned} &\min_x \sum_{k=1}^{q} w_k E[U(\tilde{c}_k x)], \\ &s.t.\ \ x \in S, \\ &\qquad w_k \in \Lambda \ \ \forall k \in \{1, \ldots, q\}, \end{aligned} \tag{3}$$

or equivalently

$$\begin{aligned} &\min_x E[U(\sum_{k=1}^{q} w_k \tilde{c}_k x)], \\ &s.t.\ \ x \in S, \\ &\qquad w_k \in \Lambda \ \ \forall k \in \{1, \ldots, q\}, \end{aligned} \tag{4}$$

where $\Lambda = \{w_k : \sum_{k=1}^{q} w_k = 1, w_k \geq 0 \ \ \forall k \in \{1, \ldots, q\}\}$.

Theorem 1. *[9] A point $x^* \in S$ is an efficient solution to problem (2) if and only if $x^* \in S$ is optimal for problem (4).*

Given that the random variable $F(x, \tilde{c}) = \sum_{k=1}^{q} w_k \tilde{c}_k x$ in (4) is a linear function of the random objectives $\tilde{c}_k x$; its variance depends on the variances of $\tilde{c}_k x$ and on their covariances. Since each $\tilde{c}_k x$ follows a normal distribution with mean μ_k and covariance σ_k^2, the function $F(x, \tilde{c})$ follows a normal distribution with mean μ and covariance σ^2 where,

$$\mu = \sum_{k=1}^{q} \mu_k = \sum_{k=1}^{q} w_k \bar{c}_k x, \tag{5}$$

$$\sigma^2 = \sum_{k=1}^{q} w_k^2 \sigma_k^2 + 2 \sum_{k,s=1}^{q} w_k w_s \sigma_{ks}, \tag{6}$$

where σ_{ks} denotes the covariance of the random objectives $\bar{c}_k x$ and $\bar{c}_s x$. Finally, we obtain the following quadratic problem:

$$\min_x \sum_{k=1}^{q} w_k \bar{c}_k^t x - \frac{a}{2} \left(\sum_{k=1}^{q} w_k^2 \sigma_k^2 + 2 \sum_{\substack{k,s=1 \\ k<s}}^{q} w_k w_s \sigma_{ks} \right), \quad s.t. \;\; x \in S, \tag{7}$$

or

$$\min_x \sum_{k=1}^{q} w_k \bar{c}_k^t x - \frac{a}{2} \left(\sum_{k=1}^{q} w_k^2 x^t V_k x + 2 \sum_{\substack{k,s=1 \\ k<s}}^{q} w_k w_s x^t V_{ks} x \right), \quad s.t. \;\; x \in S, \tag{8}$$

where $\bar{c}_k = (\bar{c}_{k1}, \bar{c}_{k2}, ..., \bar{c}_{kn})$ is the k-th component of the expected value of the random multinormal vector $\tilde{c}$, V_{ks} and V_k are elements of the positive definite covariance matrix V of $\tilde{c}$:

$$V = \begin{pmatrix} V_1 & V_{12} & ... & V_{1s} & ... & V_{1q} \\ V_{21} & V_2 & ... & V_{2s} & ... & V_{2q} \\ ... & ... & ... & ... & ... & ... \\ V_{k1} & V_{k2} & ... & V_{ks} & ... & V_{kq} \\ ... & ... & ... & ... & ... & ... \\ V_{q1} & V_{q2} & ... & V_{qs} & ... & V_q \end{pmatrix}.$$

4 Review of DC programming and DCA

A general DC program has the form:

$$\alpha = inf\{f(x) = g(x) - h(x) : x \in \mathbb{R}^n\}, \tag{9}$$

where $g,\ h$ are lower semicontinuous proper convex functions on $\mathbb{R}^n$ called DC components of the DC function f while $g - h$ is a DC decomposition of f .
The duality in DC associates to problem (9) the following dual program:

$$\alpha = inf\{h^*(y) - g^*(y) : y \in \mathbb{R}^n\}, \tag{10}$$

where g^* and h^* are respectively the conjugate functions of g and h.
The conjugate function of g is defined by:

$$g^*(y) = sup\{\langle x, y\rangle - g(x) : x \in \mathbb{R}^n\}. \tag{11}$$

From [16], the most used necessary optimality conditions for problem (9), is:

$$\emptyset \neq \partial h(x^*) \subset \partial g(x^*), \tag{12}$$

where $\partial h(x^*) = \{y^* \in \mathbb{R}^n : h(x) \geq h(x^*) + < x - x^*, y^* >, \forall x \in \mathbb{R}^n\}$ is the subdifferential of h at x^*.
A point x^* is called critical point of $g - h$ if

$$\emptyset \neq \partial g(x^*) \cap \partial h(x^*). \tag{13}$$

DCA constructs two sequences $\{x^i\}$ and $\{y^i\}$ (candidates for being primal and dual solutions, respectively), such that their corresponding limit points satisfy the local optimality conditions (12) and (13). There are two forms of DCA: the simplified DCA and the complete DCA. In practice, the simplified DCA is most used than the the complete DCA because it is less expensive [14]. The simplified DCA has the following scheme [14, 19]:

Simplified DCA Algorithm
Step 1 : Let $x^0 \in \mathbb{R}^n$ given. Set $i = 0$.
Step 2 : Calculate $y^i \in \partial h(x^i)$.
Step 3 : Calculate $x^{i+1} \in \partial g^*(y^i)$.
Step 4 : If a convergence criterion is satisfied, then **stop**, else set $i = i + 1$ and **goto** step 2.

We also can note that: [16, 19]

- DCA is a descent method without linesearch.
- If $g(x^{i+1}) - h(x^{i+1}) = g(x^i) - h(x^i)$, then x^i is a critical point of f and y^i is a critical point of $h^* - g^*$.
- DCA has a linear convergence for general DC programs, and has a finite convergence for polyhedral programs.
- If the optimal value of problem (8) is finite and the sequences $\{x^i\}$ and $\{y^i\}$ are bounded then every limit point x (resp. y) of the sequence $\{x^i\}$ (resp. $\{y^i\}$ is a critical point of $g - h$ (resp. $h^* - g^*$)

5 DCA Applied to Problem (8)

The function $f(x) = \min_x \sum_{k=1}^{q} w_k \bar{c}_k x - \frac{a}{2}\left(\sum_{k=1}^{q} w_k^2 \sigma_k^2 + 2 \sum_{\substack{k,s=1 \\ k<s}}^{q} w_k w_s \sigma_{ks}\right)$
in problem (8) will be decomposed in order to obtain a DC program of the form:

$$\min\{f(x) = g(x) - h(x) : x \in S\}, \tag{14}$$

with

$$g(x) = \chi_S(x) + \sum_{k=1}^{q} w_k \bar{c}_k^t x,$$

where $\chi_S(.)$ is the indicator function of the set S.

and

$$h(x) = \frac{a}{2} \left(\sum_{k=1}^{q} w_k^2 x^t V_k x + 2 \sum_{\substack{k,s=1 \\ k<s}}^{q} w_k w_s x^t V_{ks} x \right).$$

After that, we will compute the two sequences $\{x^i\}$ and $\{y^i\}$ defined as follows:
$y^i \in \partial h(x^i)$ *and* $x^{i+1} \in \partial g^*(y^i)$.

Computation of y^i:
We choose $y^i \in \partial h(x^i) = \{\nabla h(x^i)\}$.
It is equivalent to calculate:

$$y^i = a \left(\sum_{k=1}^{q} w_k^2 V_k x^i + 2 \sum_{\substack{k,s=1 \\ k<s}}^{q} w_k w_s V_{ks} x^i \right). \quad (15)$$

Computation of x^i:
We can choose $x^{i+1} \in \partial g^*(y^i)$ as the solution of the following convex problem

$$\min \left\{ \sum_{k=1}^{q} w_k \bar{c}_k^t x - x^t y^i \ : \ x \in S \right\}. \quad (16)$$

The solution x^i is optimal for the problem (14) if one of the following conditions is verified

$$|(g-h)(x^{i+1}) - (g-h)(x^i)| \leq \epsilon, \quad (17)$$

$$\|(x^{i+1}) - (x^i)\| \leq \epsilon. \quad (18)$$

Finally, the DC Algorithm that we can apply to problem (8) with the decomposition (14) can be described as follows:

Algorithm DCAMOSLP

Step 1 : Initialization: Let $x^0 \in \mathbb{R}^n$, $\epsilon, k, w \in \mathbb{R}^+$,$a > 0$, V, A, b, $\bar{c}$ given. Set $i = 0$.
Step 2 : Calculate $y^i \in \partial h(x^i)$ using (15).
Step 3 : Calculate $x^{i+1} \in \partial g^*(y^i)$, solution of the convex problem (16).
Step 4 : If one of the conditions (17) or (18) is verified, then **stop** x^{i+1} is optimal for (14), else set $i = i + 1$ and **goto** step 2.

---

6 Experimental Results

To demonstrate the performances of our algorithm, two numerical examples will be given in this section. The first is taken from [6] to show the efficiency of the algorithm. The second example is given to present the performances of

DCAMOSLP according to the variation of certain parameters.
Let us consider the following stochastic bi-objective programming problem:

$$\begin{cases} \min\limits_{x} (\tilde{c}_{11}x_1 + \tilde{c}_{12}x_2, \tilde{c}_{21}x_1 + \tilde{c}_{22}x_2), \\ \quad s.t. \;\; x_1 + 2x_2 \geq 4, \\ \quad\quad x_1, x_2 \leq 3, \\ \quad\quad x_1, x_2 \geq 0, \end{cases} \tag{19}$$

with $\tilde{c} = (\tilde{c}_{11}, \tilde{c}_{12}, \tilde{c}_{21}, \tilde{c}_{22})^t$ being a random vector multinormal with expected value $\overline{c} = (0.5, 1, 1, 2.5)^t$ and with positive definite covariance matrix:

$$V = \begin{pmatrix} 25 & 0 & 0 & 3 \\ 0 & 25 & 3 & 0 \\ 0 & 3 & 1 & 0 \\ 3 & 0 & 0 & 9 \end{pmatrix}.$$

For this test, we will take $\epsilon = 10^{-6}$ and $x^0 = (0, 0)$ as initial point. The application of algorithm DCAMOSLP to this problem for different values of the coefficient of incurred risk a and a fixed weight vector $\mu = (0.8, 0.2)^t$ gives the results in Table 1 where *nbr_it* is the number of iterations.

Table 1. Results for different values of parameter a.

a	(x_1^*, x_2^*)	$\overline{c}_1 x^*$	$\overline{c}_2 x^*$	*nbr_it*
10^{-30}	$(3, 0.5)$	2	4.25	2
10^{-20}	$(3, 0.5)$	2	4.25	2
10^{-10}	$(3, 0.5)$	2	4.25	3
10^{-2}	$(3, 0.5)$	2	4.25	3
1	$(3, 3)$	4.5	10.5	5
10	$(3, 3)$	4.5	10.5	5
10^2	$(3, 3)$	4.5	10.5	5

The non dominated solution $(3, 0.5)$ is obtained for values of parameter $a \leq 10^{-2}$. The non dominated solution for $w = (0.8, 0.2)^t$ in Ref. [6] is $(3, 0.5)$. We also note that the number of iterations decreases with the decrease of the parameter a.

Now we will test the performance of the algorithm with a second problem which has a larger set of feasible solutions.

$$\begin{cases} \min\limits_{x} (\tilde{c}_{11}x_1 + \tilde{c}_{12}x_2, \tilde{c}_{21}x_1 + \tilde{c}_{22}x_2), \\ \quad s.t. \;\; 2x_1 + 3x_2 \geq 10, \\ \quad\quad x_1, x_2 \leq 5, \\ \quad\quad x_1, x_2 \geq 0, \end{cases} \tag{20}$$

with $\overline{c} = (6, -5, 3, 8)^t$ and positive definite covariance matrix:

$$V = \begin{pmatrix} 14 & 0 & 0 & 3 \\ 0 & 12 & 3 & 0 \\ 0 & 3 & 2 & 0 \\ 3 & 0 & 0 & 8 \end{pmatrix}.$$

The results of application of algorithm DCAMOSLP to this problem for different values of parameter a and the weight vector w are given in Table 2.

Table 2. Results for different values of a and vector w.

a	w	(x_1^*, x_2^*)	nbr_it
	$(0.2, 0.8)$	$(0.5524, 2.9651)$	2
	$(0.8, 0.2)$	$(0, 5)$	2
10^{-20}	$(0.6, 0.4)$	$(0, 3.3333)$	2
	$(0.5, 0.5)$	$(0, 3.3333)$	2
	$(0.9, 0.1)$	$(0, 5)$	2
	$(0.2, 0.8)$	$(0.5506, 2.9663)$	5
	$(0.8, 0.2)$	$(0, 5)$	2
10^{-10}	$(0.6, 0.4)$	$(0, 3.3333)$	3
	$(0.5, 0.5)$	$(0, 3.3333)$	3
	$(0.9, 0.1)$	$(0, 5)$	2
	$(0.2, 0.8)$	$(0, 3.3333)$	5
	$(0.8, 0.2)$	$(0, 5)$	3
10^{-2}	$(0.6, 0.4)$	$(0, 3.3333)$	4
	$(0.5, 0.5)$	$(0, 3.3333)$	3
	$(0.9, 0.1)$	$(0, 5)$	3
	$(0.2, 0.8)$	$(5, 5)$	5
	$(0.8, 0.2)$	$(5, 5)$	4
10	$(0.6, 0.4)$	$(5, 5)$	4
	$(0.5, 0.5)$	$(5, 5)$	4
	$(0.9, 0.1)$	$(5, 5)$	5
	$(0.2, 0.8)$	$(5, 5)$	4
	$(0.8, 0.2)$	$(5, 5)$	5
10^2	$(0.6, 0.4)$	$(5, 5)$	5
	$(0.5, 0.5)$	$(5, 5)$	5
	$(0.9, 0.1)$	$(5, 5)$	5

We observe from the results that the algorithm DCAMOSLP gives efficient solutions of the multiobjective stochastic problem for small values of the coefficient of incurred risk ($a \leq 10^{-2}$). The number of iterations decreases with the decrease of the parameter a.

7 Conclusion

We have presented a DC optimization approach for solving a multiobjective stochastic problem with multivariate normal distributions in which the objective functions should be minimized. The experimental results show the efficiency of the algorithm. However further experimental validation of this observation and comparison with existing methods is needed. As future works, an algorithm for a stochastic multiobjective maximization problem is planned.

References

1. Alarcon-Rodriguez, A., Ault, G., Galloway, S.: Multiobjective Planning of Distributed Energy Resources Review of the State-of-the-Art. Renewable and Sustainable Energy Reviews **14**(5), 1353-1366 (2010)
2. Ben Abdelaziz, F., Mejri, S.: Application of Goal Programming in a Multi-objective Reservoir Operation Model in Tunisia. European Journal of Operational Research **133**, 352-361 (2001)
3. Ben Abdelaziz, F., Lang, P., Nadeau, R.: Distributional Unanimity in Multiobjective Stochastic Linear Programming. J. Clmaco (ed.), Multicriteria Analysis Springer-Verlag Berlin Heidelberg 1997
4. Ben Abdelaziz, F., L'efficacité en programmation multi-objectifs stochastique. Ph.D. Thesis, Université de Laval, Québec,(1992)
5. Bravo, M., Gonzalez, I.: Applying Stochastic Goal Programming: A Case Study on Water Use Planning. European Journal of Operational Research. **2**(196), 1123-1129 (2009)
6. Caballero, R., Cerd, E., del Mar Muoz, M., and Rey, L.: Stochastic approach versus multiobjective approach for obtaining efficient solutions in stochastic multiobjective programming problems. European Journal of Operational Research, **158**(3), 633-648 (2004)
7. Caner, T.Z., Tamer, U.A.: Tactical Level Planning in Float Glass Manufacturing with Co- Production, Random Y ields and Substitutable Products. European Journal of Operational Research. **199**(1), 252-261 (2009)
8. Fazlollahtabar, H., Mahdavi, I.: Applying Stochastic Programming for Optimizing Production Time and Cost in an Automated Manufacturing System. In: International Conference on Computers & Industrial Engineering, 1226-1230, Troyes 6-9 July (2009)
9. Geoffrion, A.M.: Proper Efficiency and the Theory of Vector Maximimization. Journal of Mathematical Analysis and Applications **22**(3), 618-630 (1968)
10.
11. Goicoechea, A., Dukstein, L., Bulfin, R.T.: Multiobjective Stochastic Programming. the PROTRADE-method. Operation Research Society of America (1976)
12. Klein, G., Moskowitz, H., Ravindran, A.: Interactive multiobjective optimization under uncertainty. Management Science. **36**(1), 58-75 (1990)
13. Kumral, M.: Application of Chance-Constrained Programming Based on Multiobjective simulated Annealing to Solve Mineral Blending Problem. Engineering Optimization. **35**(6), 661-673 (2003)
14. Le Thi, H.A., Pham Dinh, T.: Solving a class of linearly constrained indefinite-quadratic problems by dc algorithms. Journal of Global Optimization **11**(3), 253-285 (1997b)

15. Le Thi, H.A., Pham Dinh, T.: A continuous approach for globally solving linearly constrained quadratic zero-one programming problems. Optimization **50**, 93-120 (2001)
16. Le Thi, H.A., Pham Dinh, T.: The dc (difference of convex functions) programming and DCA revisited with DC models of real world nonconvex optimization problems. Annals of Oper. Res. **133**, 23-46 (2005)
17. Le Thi, H.A., Pham Dinh, T., Huynh, V.N.: Exact penalty and error bounds in DC programming. J. of Glob. Opt. **52**, 509-535 (2012)
18. Le Thi, H.A., Pham Dinh, T., Nguyen, C.N., Nguyen, V.T.: DC programming techniques for solving a class of nonlinear bilevel programs. J. of Glob. Opt. **44**, 313-337 (2009)
19. Pham Dinh, T., Le Thi, H.A.: Convex analysis approach to DC programming: Theory, Algorithms and Applications (dedicated to Professor Hoang Tuy on the occasion of his 70th birthday). Acta Mathematica Vietnamica **22**, 289-355 (1997a)
20. Pham Dinh, T., Nguyen, C.N., Le Thi, H.A.: DC Programming and DCA for Globally Solving the Value-At-Risk, Comput. Manag. Sci. **6**, 477-501 (2009)
21. Teghem, J., Kunsch, P.: Application of Multiobjective Stochastic Linear Programming to Power Systems Planning. Engineering Costs and Production Economics **9**(13), 83-89 (1985)
22. Teghem, J., Dufrane, D., Thauvoye, M. and Kunsch, P.L.: Strange, an Interactive Method for Multiobjective Stochastic Linear Programming under Uncertainty. European Journal of Operational Research **26**(1), 65-82 (1986)
23. Vahidinasab, V., Jadid, S.: Stochastic Multiobjective Self-Scheduling of a Power Producer In Joint Energy & Reserves Markets. Electric Power Systems Research **80**(7), 760-769 (2010)
24. Wang, Z., Jia, X.P., Shi, L.: Optimization of Multi-Product Batch Plant Design under Uncertainty with Environmental Considerations. Clean Technologies and Environmental Policy **12**(3), 273-282 (2009)