# *Stochastic evaluation of defects in eddy currents non-destructive testing system*

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***Abstract*— In this paper, we discuss the defect evaluation in a conductive material, by a method based on stochastic finite elements. This later allows then to obtain a hazard distribution of one or more physical properties of the material. To do this, we have considered two kinds of distributions of random variables; Gaussian and Lognormal ones. Those variables will represent the distribution of the electrical conductivity in the area of the defect to be investigated and an evaluation of the impact on the reliability of the device will be done.**

**A stochastic finite element calculation code is developed and the variation of the impedance is then deduced by doing a scan along the surface of the conductive plate. The computational was performed in the two cases successively, Gaussian and Lognormal distributions. A post-treatment is performed by the calculation of the reliability index. This results Allowed us to evaluate the probability of failure associated with the nature of the fault present on the conductive plate for both cases of random variables.**

**The study is completed by the evaluation of the impact of the defect thickness on the reliability index using lognormal distribution of the random variable. The results obtained appear to be interesting.**

**Keywords—** *Stochastic finite elements, lognormal, Gaussian, random variable, default, reliability index, probability of failure*.

## I. INTRODUCTION

The difficulty encountered when solving a practical problems is the capability to identifies the physical properties such as electrical conductivity, magnetic permeability which should be known prior to the treatment of the problem. In this case, in most cases study, the input data are often searched by help of experiment or by using them with some uncertainty.

Nowadays, researchers are increasingly focusing on modeling in uncertain environments using stochastic calculus.

The present work is concerned with the study of uncertainties that appear in the inspection and evaluation of defects in materials. Uncertainty concerns the estimation of physical properties for the conduct of a simulation or numerical modeling study for the detection or characterization of defects in materials. The physical property distribution concerns the electrical conductivity which is considered at first as Gaussian random variable behavior in the defect area. In order to study the best way to represent the stochastic distribution of the electrical conductivity a lognormal distribution is then considered and applied in the current study.

Probabilistic process is managed by a series expansion of the electrical conductivity through the Hermit polynomials of a Gaussian reduced centered variable.

The electromagnetic model of the studied system is the 2D magnetodynamic equation in axisymmetric coordinates. The physical property distribution thus obtained is developed on series of polynomial chaos.

The obtained after resolution of the stochastic algebraic system informs us at first of the existence of the defect through the variation of the impedance related to the sensor dedicated to the control by eddy current.

The second step concerns the post-processing where the reliability index and the probability of failure are calculated and a comparison of these parameters is carried out for the two types of random variables which are the Gaussian and lognormal respectively [1-3].

The final step of this study is devoted to evaluating the reliability index in the defect zone by considering three thicknesses by taking the reference thickness that used in the second step. The Application chosen has been studied for the validation of the stochastic finite elements model in previous work [1].

## STOCHASTIC ELECTROMAGNETIC MODEL

### *A. Deterministic Electromagnetic Model*

The electromagnetic equation is obtained from Maxwell's equations associated with those of the middle relations and the law of ohms. In case of harmonic hypothesis, the electromagnetic formulation for determinist problem is given as follows [1-7].

$$\vec{\nabla} \wedge \left( \neg \vec{\nabla} \wedge \vec{A} \right) + j \pm \mu \vec{A} = \vec{J}_S \qquad (1)$$

### *B. Random Electrical Conductivity*

The electrical conductivity is distributed as two types of random variables, the distribution coefficients of which are listed in Table 1, with the characteristics of the two random variables.

### C. Stochastic algebraic matrix

The expressions of electric conductivity and the unknown magnetic vector potential as random variables in the base of Hermit polynomials permit to write:

$$A = \sum_{j=0}^{n_A} A_j \Psi_j(\xi_1, \cdots, \xi_M) \quad (1)$$

$$\sigma = \sum_{i=0} \sigma_i \Psi_i(\xi_1) \quad (2)$$

$\{\xi_1,..,\xi_2\}$ centered reduced Gaussian variables

$n_A$ : chaos polynomial order.

The final form of the stochastic algebraic system is as follow [6]:

$$\sum_{j=0}^{p-1} (M^{s\,\prime}_{jk} + j\mu N^{s}_{jk}) A_j = F^{s}_{k} \quad (3)$$

$$M^{s\,\prime}_{jk} = d_{0jk} M_0 \quad (4)$$

$$N^{s}_{jk} = \sum_{i=0}^{p-1} d_{ijk} N^{s}_{i} \quad (5)$$

$d_{ojk}$ et $d_{ijk}$ are constants to determined [1-4].

TABLE I. STOCHASTIC DISTRIBUTION OF ELECTRICAL CONDUCTIVITY COEFFICIENTS

| Gaussian random variable (μ,σ) | *Average value* | *standard deviation* | Stochastic distribution coefficients | | |
|---|---|---|---|---|---|
| | | | $a_{00}$ | $a_{11}$ | $a_{22}$ |
| | $\mu = 0.76 MS^{-1}$ | $\sigma = 0.9$ | $0.76 MS^{-1}$ | 0.9 | 0 |
| **Lognormal random variable** (λ,ζ) | $\zeta = 0.76 MS^{-}$ | $^{1}\lambda = 0.9$ | 3.20 | 2.88 | 1.30 |

## II. APPLICATION AND RESULTS

The application chosen is a non-destructive testing device, which represents a steam generator tube in nuclear power plants subjected to eddy currents inspection [1-2].

The obtained axisymmetric stochastic model is applied for numerical simulations of a cylindrical device behavior. A reliability analysis and an estimate of the probability of failure are made. Thus the study was conducted under the 2D axisymmetric hypothesis. The device for NDT consists of a 2D approximation of a conducting cylinder whose conductivity is equal to 0.76 MS/m. The defect is characterized by a length of 10 mm and a depth of 0.75 mm. The inductor, representing the sensor, has 140 turns, it is powered by an alternating current of amplitude 0.008A and a frequency of 150 kHz.

The results obtained for the two types of random variables and the different thickness of the defect by associating the lognormal random variable, are presented by figures 1, 2, 3 and 4.

Fig. 1. Evaluation of the reliability index in the default for both types of random varibles for a standard deviation of 0.9

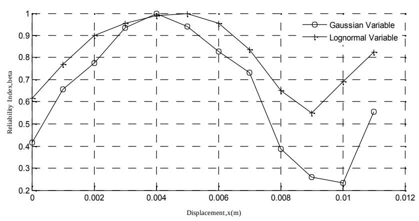



Fig. 2. Impedance variation in the default for both random variables

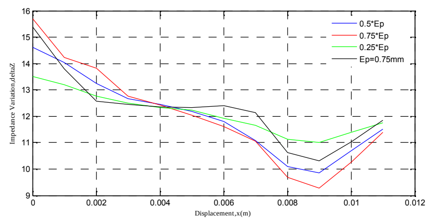



Fig. 3. impedance variation for different thickness of the defect

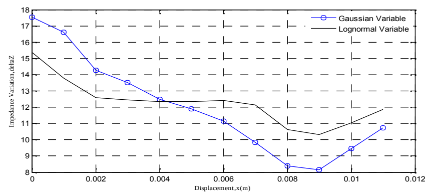
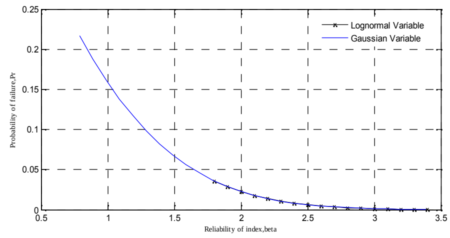



Fig. 4. Evolution of the probability of failure for both types of random variables




## CONCLUSION

The distribution of the electrical conductivity by the Gaussian random variable, informs us on the surface state of the defect, characterized by a lack of material because of certain distribution terms worth zero; Against the lognormal random variable is more realistic by a more homogeneous distribution of the electrical conductivity in the defect, which is reflected in Figures 1, 2 and 4. The variation of the thickness of the defect shows us that indeed the defect is important, the more variation of the impedance is important. The stochastic finite element model gives us very precise information about the evolution of the probability of failure by an appropriate choice of the random variable, which is lognormal for the case of our study.

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